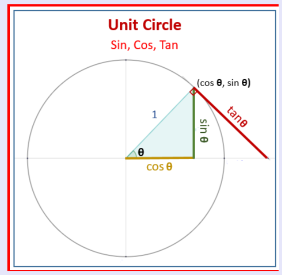
Trigonometry

Lecture 11



Feb 19-8:47 AM

Verify $\dfrac{\sin^2 x}{1+\cos x} = 1 - \cos x$

Recall

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

Recall

$A^2 - B^2 = (A+B)(A-B)$

$$\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x}$$

$$= \frac{1^2 - \cos^2 x}{1+\cos x}$$

$$= \frac{\cancel{(1+\cos x)}(1-\cos x)}{\cancel{1+\cos x}}$$

$$= \boxed{1-\cos x} \checkmark$$

Sep 12-10:25 AM

Verify $\dfrac{1+\sin x}{\cos x} = \dfrac{\cos x}{1-\sin x}$ ✓

$$\frac{1+\sin x}{\cos x} = \frac{(1+\sin x)\cdot\cos x}{\cos x\cdot\cos x}$$

$$= \frac{(1+\sin x)\cdot\cos x}{\cos^2 x} = \frac{(1+\sin x)\cdot\cos x}{1-\sin^2 x}$$

Recall

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$= \frac{\cancel{(1+\sin x)}\cdot\cos x}{\cancel{(1+\sin x)}(1-\sin x)}$$

$$= \frac{\cos x}{1-\sin x}$$

Do not do the Following

$$\frac{\cancel{1}+\cancel{\sin x}}{\cancel{1}-\sin^2 x} = \frac{1}{\sin x}$$

Sep 12-10:29 AM

Verify $\tan x + \cot x = \sec x \csc x$ ✓

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin x\cdot\sin x}{\cos x\cdot\sin x} + \frac{\cos x\cdot\cos x}{\sin x\cdot\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x\cos x}$$

$$= \frac{1}{\sin x\cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x}$$

$$= \csc x\cdot\sec x$$

Sep 12-10:35 AM

Verify $\dfrac{\cos^2 x \sin x - \cos^3 x}{2\sin^4 x - \sin^2 x} = \dfrac{\cot^2 x}{\sin x + \cos x}$ ✓

$$\frac{\cos^2 x \sin x - \cos^3 x}{2\sin^4 x - \sin^2 x} = \frac{\cos^2 x(\sin x - \cos x)}{\sin^2 x(2\sin^2 x - 1)} =$$

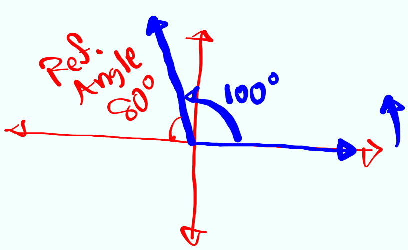
$$2\sin^2 x - 1 = 2\sin^2 x - (\sin^2 x + \cos^2 x)$$
$$= 2\sin^2 x - \sin^2 x - \cos^2 x$$
$$= \sin^2 x - \cos^2 x$$

$$= \frac{\cot^2 x(\sin x - \cos x)}{\sin^2 x - \cos^2 x} = \frac{\cot^2 x\,\cancel{(\sin x - \cos x)}}{\cancel{(\sin x - \cos x)}(\sin x + \cos x)}$$

$$= \boxed{\frac{\cot^2 x}{\sin x + \cos x}}$$

Sep 12-10:40 AM

Draw 100° in standard position, clearly label its ref. angle.

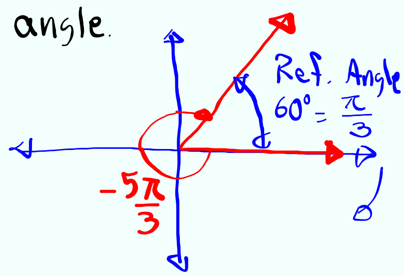


Draw $-\frac{5\pi}{3}$ in standard position, clearly label its ref. angle.

$\pi = 180^\circ$

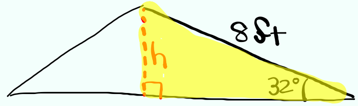
$\frac{\pi}{3} = 60^\circ$

$\frac{5\pi}{3} = 300^\circ$



Sep 12-10:50 AM

Find area of the triangle below



Area $= \frac{b \cdot h}{2}$

$\sin 32° = \frac{h}{8}$

Cross - Multiply

$h = 8 \cdot \sin 32°$

$= \frac{\cancel{12}^{6} \cdot 8 \cdot \sin 32°}{\cancel{2}}$

$= 48 \sin 32°$

$\approx 25 \text{ ft}^2$

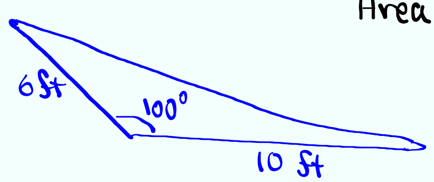
If we have 2 sides and angle between them, Area $= \frac{1}{2} bc \sin A$

Area $= \frac{1}{2} ac \sin B$

Area $= \frac{1}{2} ab \sin C$

Sep 12-10:57 AM

Find the area

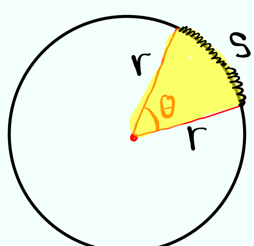


Area $= \frac{1}{2} \cdot 6 \cdot 10 \cdot \sin 100°$

$= 30 \sin 100°$

$\approx 30 \text{ ft}^2$

Central angle & Sector



Central Angle θ

Arc length S

Radius r

Area of the Sector

$A = \frac{1}{2} r^2 \theta$

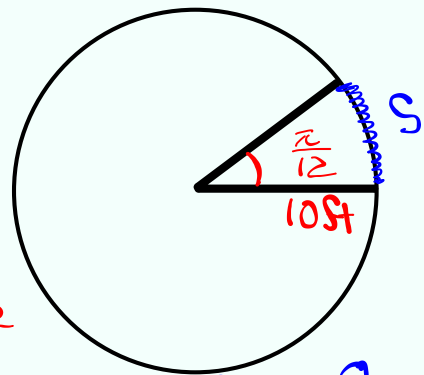
θ must be in radians

Arc length S

$S = r\theta$

Sep 12-11:03 AM

Consider a Sector with central angle $\frac{\pi}{12}$ and radius 10 ft.

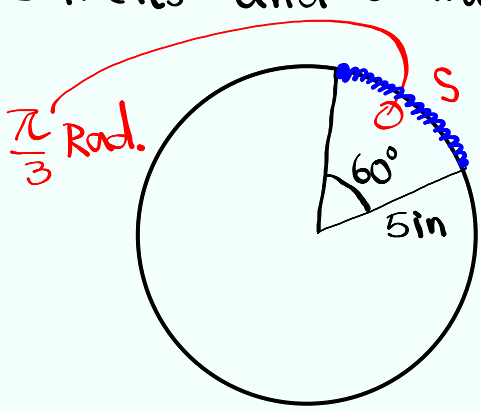


$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \cdot 10^2 \cdot \frac{\pi}{12}$$
$$= \frac{50\pi}{12} = \frac{25\pi}{6} \text{ ft}^2$$

$$S = r\theta$$
$$= 10 \cdot \frac{\pi}{12} = \frac{5\pi}{6} \text{ ft}$$

Sep 12-11:11 AM

Draw a Sector on a Circle with radius 5 inches and Central angle of 60°.



1) Find its area

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \cdot 5^2 \cdot \frac{\pi}{3} = \frac{25\pi}{6} \text{ in}^2.$$

2) Find arc length

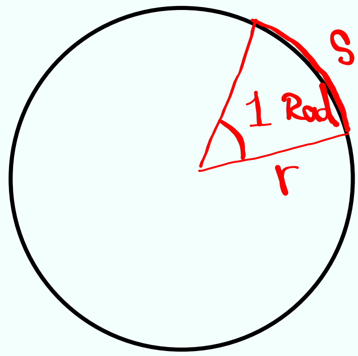
$$S = r\theta = 5 \cdot \frac{\pi}{3} = \frac{5\pi}{3} \text{ in.}$$

Sep 12-11:15 AM

What is Radian?

$$s = r\theta$$

If $s = r \rightarrow \theta = 1$ Radian.

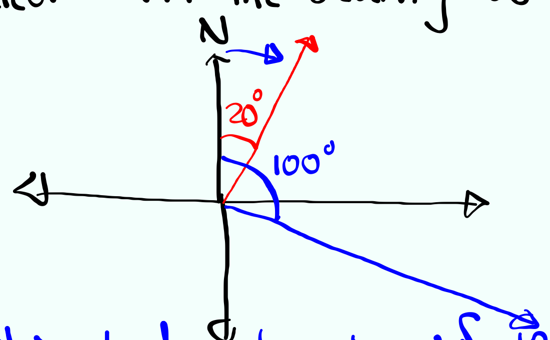


Sep 12-11:20 AM

Introduction to Bearing

Method 1: Measure from North axis, go clockwise.

Ship traveled with the bearing of 20°.

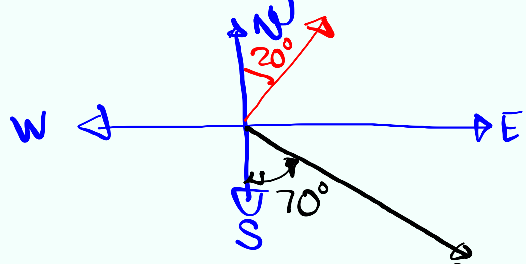


Another ship had a bearing of 100°.

Sep 12-11:22 AM

Method 2: Measure from North & South axis towards East & West

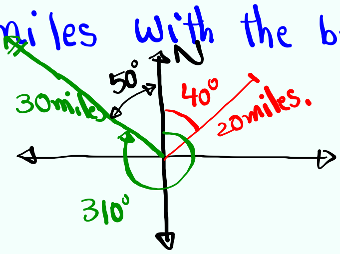
Ship had a bearing N 20° E.



another ship had a bearing of S 70° E.

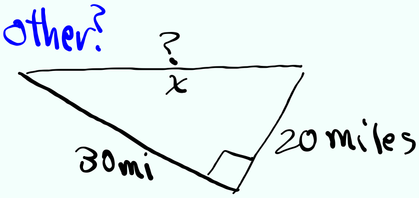
Sep 12-11:26 AM

Ship A went 20 miles with the bearing of 40°.



At the Same time,

ship B went 30 miles with the bearing of 310°. How far apart are they from each other?



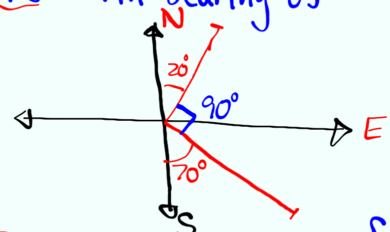
$30^2 + 20^2 = x^2$

$900 + 400 = x^2$

$1300 = x^2$ $\quad x = \sqrt{1300} \approx \boxed{36 \text{ miles}}$

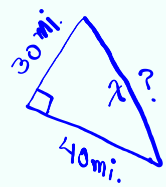
Sep 12-11:29 AM

Ship A goes 30 miles with bearing of N 20° E.



At the same time

Ship B goes 40 miles with bearig of S 70° E. How far are they apart?



$30^2+40^2=x^2$

$\vdots$

$\boxed{x=50}$

50 miles apart